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FLOW OF A PLANE JET OF LIQUID FROM A RESERVOIR WITH FLEXIBLE WALLS NEAR A SCREEN*

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The problem of the jet overhang created by a jet emerging through an orifice in a flexible barrier is considered. A numerical investigation is made of the mutual influence of the shape of the flexible reservoir walls and the jet parameters for different ratios of the pressure and distance to the screen.

The problem considered here is connected with calculations of the flow in flexible barriers of vessels on air cushions. Previous studies /1, 2/ have considered detached flow around a flexible casing near a screen, i.e., flow typical for the chamber scheme of formation of an air cushion. The study of flows in a jet scheme involves considerable computational complexity, and in this connection the problem is usually simplified by being split into two: computation of the shape of the casing on the assumption that the pressure distribution is a step function /3/, and computation of the jet flow from a nozzle device of given shape, in which context the nozzle is usually assumed to have straight walls /4, 5/. It is still not known to what degree the actual pressure distribution affects the shape of the casing, or how far the latter affects the jet parameters. The combined examination of both these problems in /6/, for the case in which the physical picture is symmetric about the vertical axis, shows that this influence, for real ratios of the width of the orifice in the casing to its length, is negligible. However, the problem when there is no symmetry remains open, in particular for large transverse pressure drops.

1. This appears is devoted to a numerical solution of the problem of a planar jet emerging from an orifice in a flexible barrier, in its exact non-linear steady-state formulation, for unequal pressures p_1 and p_0 and different casing lengths L_1 and L_2 from the edges of the orifice A and B to the attachment points A' and B' (Fig.1,a). The casing is assumed to be absolutely flexible (zero moment), weightless and inextensible; the liquid is assumed to be weightless, inviscid and incompressible. The casing is attached at its ends A' and B' to the vertical walls of the channel, and the ends A and B are assumed to be connected by a thin thread that does not obstruct the motion of the flow. The thread thereby keeps the ends of the

*Prikl. Matem. Mekhan., 54, 1, 34-39, 1990 casing at a tiven distance β and corresponds to a segment of the common tangent to the casing at the points A and B.

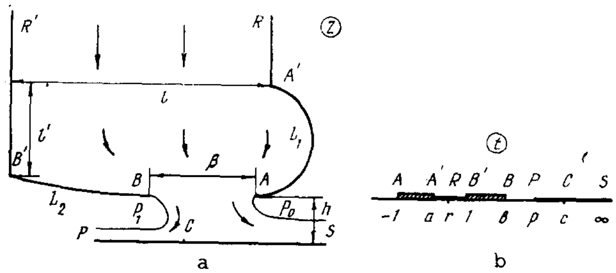

Fig.1

Under these conditions, the problem reduces to determining an analytic function satisfying boundary conditions of a special type. The domain corresponding to the flow in the physical plane Z is mapped conformally onto the upper half-plane of the complex variable $t = x + iy$ (for the correspondence of the points see Fig.1,b). The complex potential is

$$W(t) = \frac{Q}{\pi}\frac{r-p}{r-c}\int\limits_{-1}^{t}\frac{t-c}{(t-r)(t-p)}\,dt \tag{1.1}$$

where Q is the liquid discharge in the jet. The boundary conditions for the Joukowsky function

$$\omega = i\ln\frac{1}{v_0}\frac{dW}{dZ} = \theta + i\ln\frac{v}{v_0}$$

(v, θ are the absolute value of the velocity vector and its inclination to the horizontal axis, v_0 the velocity on the free surface AS) are written as

$$\operatorname{Re}\omega(x) = \theta(x) = \begin{cases} -\pi/2, & a < x < 1 \\ -\pi, & p < x < c, \\ 0, & c < x < \infty \end{cases} \qquad \operatorname{Im}\omega(x) = \begin{cases} 0, & -\infty < x < -1 \\ \ln(v_1/v_0), & b < x < p \end{cases} \tag{1.2}$$

Here v_1 is the velocity of the liquid on the free surface BP (according to Bernoulli's equation $v_1 = \sqrt{v_0^2 - 2(p_1 - p_0)/\rho}$).

On the sections of the boundary corresponding to the flexible casing, the imaginary and real parts of the function ω satisfy the Laplace condition:

$$\begin{aligned} \frac{d\theta}{ds} &= \frac{\rho v_0^2}{2T}\left(1 - \left(\frac{v}{v_0}\right)^2\right), \quad x \in (-1, a) \\ \frac{d\theta}{ds} &= -\frac{\rho v_0^2}{2T}\left(\left(\frac{v_1}{v_0}\right)^2 - \left(\frac{v}{v_0}\right)^2\right), \quad x \in (1, b) \end{aligned} \tag{1.3}$$

Here ρ is the density of the liquid, T is the tension of the casing (which is the same at all points), and s is the arc abscissa measured from the points A and B, respectively.

2. The boundary-value problem (1.2), (1.3) can be solved by various numerical or numerical-analytical methods, such as that described in /7/. The best method to be used here is the generalized Levi-Civita method /2/.

The function $\omega(t)$ is sought in the form of a sum

$$\omega(t) = \omega_0(t) + \omega_1(t) + \omega_2(t) \tag{2.1}$$

The function $\omega_0(t)$ satisfies conditions (1.2); in the intervals $[-1, a]$ and $[1, b]$ its real part has constant values, equal to the limiting values of $\theta(x)$ for motion along the casing to the attachment points A' and B' $(\theta_1 = \operatorname{Re}\omega(1+0),\ \theta_2 = \operatorname{Re}\omega(a-0))$; the function can be represented by means of a modification of the Keldysh-Sedov formula /8/:

$$\omega_0(t) = \theta_1 + \frac{1}{i\pi}\int_{-1}^{t}\left\{\frac{B_a}{(t+1)(t-a)} + \frac{B_1}{(t+1)(t-1)} + \frac{B_c}{(t-p)(t-c)} + \right. \tag{2.2}$$

$$\left.\frac{{}^1/_2F - B_a - B_1 - B_c - (b+1)\,\partial F/\partial b}{(t+1)(t-p)}\right\} g_2(t)\,dt + \frac{1}{i\pi}\frac{\partial F}{\partial b}\int_{-1}^{t}\frac{dt}{g_1(t)}$$

$$B_a = \frac{a+1}{|g_2(a)|}\left(\frac{\pi}{2}+\theta_1\right), \quad B_1 = -\frac{2}{|g_2(1)|}\left(\frac{\pi}{2}+\theta_2\right), \quad B_c = -\frac{c-p}{|g_2(c)|}\pi$$

$$F = \theta_1 I(-1, a) - \frac{\pi}{2} I(a, 1) + \theta_2 I(1, b) + \ln\frac{v_1}{v_0} I(b, p) - \pi I(p, c)$$

$$g_1(t) = \sqrt{\frac{(t-b)(t-p)}{t+1}}, \quad g_2(t) = \sqrt{\frac{(t+1)(t-p)}{t-p}}, \quad I(\alpha, \beta) = \int_{\alpha}^{\beta}\frac{dx}{|g_2(x)|}$$

The real parts of the functions $\omega_1(t)$ and $\omega_2(t)$ must vanish in the intervals $(a, 1)$, (p, c), (c, ∞), as do their imaginary parts in the intervals $(-\infty, -1)$ and (b, p). In addition, it is required that the real part of ω_1 vanish for $x \in (1, b)$, and that of ω_2 for $x \in (-1, a)$. Then these functions can be expressed as

$$\omega_k(t) = ig_k(t)\varphi_k(t), \quad k = 1, 2 \tag{2.3}$$

where $\varphi_1(t)$ and $\varphi_2(t)$ are functions taking real values over the entire real axis, with the exception of the intervals $(-1, a)$ and $(1, b)$, respectively.

To construct $\varphi_1(t)$, we map the upper half of the t-plane, conformally onto the upper half of the unit disc in the ζ_1-plane, in such a way that the segment $[-1, a]$ is carried into the semicircle. This mapping is accomplished by the transformations

$$t_1 = (2t - a + 1)/(a + 1), \quad \zeta_1 = -t_1 + \sqrt{t_1^2 - 1}$$

Then, according to the Levi-Civita method, the function $\varphi_1(t(\zeta_1))$ may be sought as a power series with real coefficients. Substituting $t(\zeta_1)$ into (2.3) and reducing, we obtain

$$\omega_1(t(\zeta_1)) = \tag{2.4}$$

$$\sqrt{\zeta_1 + \frac{\zeta_1^2+1}{2}\,\frac{a+1}{2b-a+1}}\sqrt{\zeta_1 + \frac{\zeta_1^2+1}{2}\,\frac{a+1}{2p-a+1}}\,\frac{1}{\sqrt{\zeta_1}}\sum_{m=0}^{\infty} c_m\zeta_1^m$$

Similarly, to construct $\varphi_2(t)$ we map t onto the half-disc in the ζ_2 plane so that the segment $[1, b]$ is carried into the semicircle. Again applying the Levi-Civita method, we write

$$\omega_2(t(\zeta_2)) = \tag{2.5}$$

$$i\sqrt{\zeta_2 + \frac{\zeta_2^2+1}{b}\,\frac{b-1}{-3-b}}\sqrt{\zeta_2 + \frac{\zeta_2^2+1}{2}\,\frac{b-1}{2p-b-1}}\,\frac{1}{\sqrt{\zeta_2}}\sum_{m=0}^{\infty} d_m\zeta_2^m$$

The real coefficients c_m and d_m are determined from conditions (1.3), which, after the substitution

$$ds = -\frac{Q}{\pi v}\,\frac{(r-p)(x-c)}{(r-c)(x-r)(x-p)}\,dx, \quad \lambda = \frac{\rho Q v_0}{\pi T}\,\frac{r-p}{r-c}$$

become

$$\frac{d}{dx}\operatorname{Re}\omega_k(x+i0) = \Lambda_k\frac{x-c}{(x-r)(x-p)}, \quad x \in (\alpha_k, \beta_k) \tag{2.6}$$

$$\Lambda_1 = -\frac{\lambda}{2}\left(\frac{v_0}{v} - \frac{v}{v_0}\right), \quad \Lambda_2 = \frac{\lambda}{2}\left(\frac{v_1^2}{vv_0} - \frac{v}{v_0}\right)$$

$$(\alpha_1 = -1,\ \beta_1 = a,\ \alpha_2 = 1,\ \beta_2 = b,\ v = v_0\exp\{\operatorname{Im}(\omega_0 + \omega_1 + \omega_2)\})$$

The function $\omega(t)$ must be bounded as $t \to \infty$; this condition leads to the equality

$$\frac{2}{\pi}\frac{\partial F}{\partial b} + \frac{c_0\sqrt{a+1}}{\sqrt{(2b-a+1)(2p-a+1)}} - \frac{d_0\sqrt{b-1}}{\sqrt{(3+b)(2p-b-1)}} = 0 \tag{2.7}$$

The problem is solved for given geometrical parameters L_1, L_2, l, l', h, β (Fig.1,a). Hence the following equations must hold:

$$\int_A^{A'} |dZ| = L_1, \quad \int_B^{B'} |dZ| = L_2, \quad \int_{A'}^{B'} dZ = -l - il'$$

$$\operatorname{Im}\int_A^C dZ = -h, \quad \exp(-i\theta_A)\int_A^B dZ = \beta + i0 \tag{2.8}$$
$$\left(\theta_A = \theta(-1), \quad dZ = \frac{Q}{\pi v_0}\frac{r-p}{r-c}\exp(i\omega)\frac{t-c}{(t-r)(t-p)}\,dt\right)$$

The condition that A and B are connected by a thin thread which does not obstruct the liquid flow may be written as

$$\theta(-1) = \theta(b) - \pi \tag{2.9}$$

Conditions (2.6), written for a finite number of collocation points x_m, together with Eqs.(2.7)-(2.9) and taking (2.2) into account, form a closed system of equations, which can be solved by a modified Newton method for the unknown constants c_m $(m = 0, \ldots, N)$, d_m $(m = 0, \ldots, N)$, $a, b, c, p, r, \lambda, \theta_1, \theta_2, Q$.

Thus, the solution of problems using the method proposed here ensures exact observance of the boundary conditions (1.2) and approximate satisfaction of conditions (1.3), with order of approximation dependent on the number N of terms retained in the series. For the computations reported below, done with 0.1-0.5% accuracy, the number N did not exceed 7-12.

3. The shape of the casing obtained by solving the problem in its exact formulation should be compared with the approximate solution, which is based on the assumption that the velocity of liquid flow inside the casing is zero. Then AA' and BB' are arcs of circles with radii R_1 and R_2. In this case the radii, angles $\theta_1, \theta_2, \theta_A, \theta_B$ and tension T may be found from the system of equations

$$R_1(\theta_1 - \theta_A) = L_1, \quad R_2(\theta_B - \theta_2) = L_2, \quad \theta_A = \theta_B - \pi \tag{3.1}$$
$$x_A - x_B = \beta\cos\theta_B, \; y_A - y_B = \beta\sin\theta_B, \; R_1 v_0^2 = R_2 v_1^2 = T/\rho$$

where x_A, y_A, x_B, y_B are the coordinates of A and B, determined from the formulae

$$x_A = l - R_1(\sin\theta_A - \sin\theta_1), \quad y_A = l' + R_1(\cos\theta_A - \cos\theta_1)$$
$$x_B = R_2(\sin\theta_B - \sin\theta_2), \quad y_B = -R_2(\cos\theta_B - \cos\theta_2)$$

(the origin is assumed to be at B').

4. The results of the computation are shown in Fig.2, where

$$\sigma = (v_1/v_0)^2, \; x' = x_B/(l-\beta), \; y' = -y_B/(l-\beta) + 0.5,$$
$$\beta' = \beta/(l-\beta)$$

and it is assumed that the geometrical parameters have the following relations to one another:

$$l' = 0, \; (L_1 + L_2)/(l-\beta) = 1.5, \quad L_1/(l-\beta) = 0.473$$

These relations were chosen so as to ensure that the slope θ_B in the approximate solution (3.1) would vanish at $\sigma = 0.1$, and so that the casing configuration would remain unchanged for all β. The solid curves correspond to the approximate solution (3.1), the dashed ones to the exact solution at $\beta' = 0.3$. It can be seen that the deviation of the shape of parts of the casing from that of arcs of circles isnegligible(of the order of 1-3%), even for orifice dimensions β comparable with L_1 and L_2. This may be attributed to the high degree of stability, and hence the low mobility, of the casing, when turns its concave surface towards the oncoming flow (according to the "sail" scheme of /9/).

Thus, even in the case of an asymmetric casing, the approximate computation is fully justified and may be used for practical purposes (in flexible barriers of vessels on air cushions the value of β' is usually at most 0.1-0.2).

The value of the gap h in problems involving a jet impinging on a screen is an independent (preassigned) parameter and, other things being equal, determines the flow regime of the jet, which may be characterized by the jet separation coefficient $k = (c-p)/(c-r)$, which is equal to the incoming flow to the region with high pressure p_1 divided by the total discharge Q. If $k = 0$ $(c = p)$ the entire jet flows to the right, and the gap $h = h_0$ depends on σ and the geometric parameters β, L_1, L_2, l, l'.

The results shown in Fig.2 for the exact solution correspond to $h = h_0(\sigma)$, i.e., $k = 0$. At $h < h_0$ the jet emerging from the nozzle is unable, under the action of the transverse pressure, to swing fully to the right before impinging on the screen; when this occurs, therefore, it separates into two jets which flow along the screen in opposite directions (Fig.1,a).

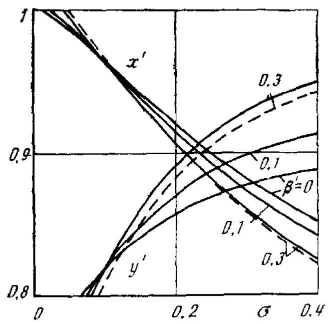

Fig.2

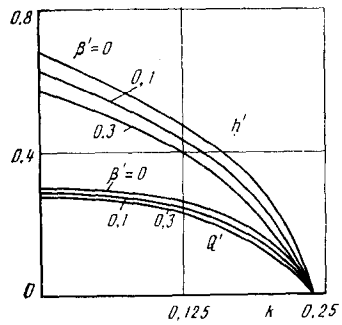

Fig.3

At $h > h_0$ the jet overhang is slightly raised, and beneath it there appears a jet flowing from left to right (from the high-pressure region p_1); the discharge in this jet increases as $h - h_0$ increases. The parameters of the jet overhang itself (e.g., the discharge Q, and the pressure at the walls and nozzle section) for a nozzle with straight walls depend rather weakly on h at $h \geqslant h_0$ /5/.

These properties of a jet impinging on a screen are consequences of conservation laws and are valid for jets emerging from various nozzle devices with straight or curved walls, in particular, in the case of the flexible nozzle considered here.

The parameters of the jet overhang for a nozzle with flexible walls at $h \leqslant h_0$ are shown in Fig.3, compared with the computed results for a jet issuing from a nozzle with straight walls set at the same angles θ_A and θ_B as the end sections of the casing A and B in the approximate solution. Fig.3 shows curves representing the relative parameters: the gap $h' = h/\beta$ and the discharge $Q' = Q/(\beta v_0)$ as a function of the jet separation coefficient k. The geometrical parameters were taken to be the same as before, $\sigma = 0.1$. As $\beta' \to 0$ the relative parameters Q' and h' for the flexible casing approach those of a straight-walled nozzle. At $\beta' = 0.3$ the discharge falls by 10-20%, and the gap by 20-30%. The reason is that the velocity vector of the liquid emerging from the nozzle, avaraged over its entire cross-section, is forced to rotate because of the asymmetric distortion of the nozzle walls.

Thus, in practical computations of jet overhangs for $\beta' > 0.1$ one must take the real configuration of the casing into consideration. As shown above, the shape of the casing may be computed approximately. However, replacing the Laplace Eqs.(1.3) by the conditions $d\theta/ds = R_k$ does not simplify the solution algorithm. It is therefore necessary to use the exact formulae (2.2)-(2.5) for the jet parameters.

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